

MAT3106

Vector Calculus and  
Mathematical Modelling of  
Fluid Flows

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Faculty of Sciences

Introductory Book

Semester 2, 2008

Published by

The University of Southern Queensland  
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# Course Specifications

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Course specifications detail what the university agrees to provide in the offering of the course and outlines the expectations of the student.

- Examiners are entitled to expect that students have read, understood and are familiar with the course specification for the courses in which they are enrolled.
- Students are entitled to expect that the course will be delivered and assessed as in the course specification.

Course specifications are available online from the USQ website

*<http://www.usq.edu.au/course/specification/2008/>*.

Always check the website for the latest version.

**Important note:** from 2008, course specifications are no longer included in printed introductory materials. The online version will ensure students access the latest version. Any updates to the course specification after the start of semester will be communicated to students.



# Required Resources

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## Study Book

The study book is very essential for this course for both external and on campus students as it contains a large part of the study material and many clarifying notes to the textbooks used. Please see the course specification for details of the textbooks.

## Email and Internet access

It is essential that students enrolled in this course have reliable access to email and the Internet and establish regular contact with the lecturer at their earliest opportunity. External students are required to communicate with the lecturer on a weekly basis (while email is a preferable option, fax and phone exchanges can be used as well, see “Lecturer’s contacts” below). It is planned that updates of the course Study Book will be published regularly on the course web page at <http://www.sci.usq.edu.au/courses/mat3106> which students should check at least once a week.

## Software

Access to MATLAB and MAPLE software is desirable but not essential to successfully complete this course. Other similar computer algebra software and packages with built in graphics can be used as an alternative.

# Student support

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## Types of enquiries

You have access to a wide variety of support services at USQ. Follow the details below or visit the “Current Students” website at <http://www.usq.edu.au/currentstudents/default.htm> for more information.

### General enquiries

USQAssist is the most efficient method for requesting assistance for:

- administrative queries
- assignment submissions
- study assistance
- contacting your lecturer.

### Technical enquiries

Enquiries relating to access to *USQConnect*, the *USQStudyDesk*, or other technical issues can also be directed to *USQAssist*. If you cannot access *USQAssist*, contact the Student IT HelpDesk on +61 7 4631 1510 or email [usqconnect@usq.edu.au](mailto:usqconnect@usq.edu.au) for assistance. The Student IT HelpDesk is staffed weekdays between 8.00am and 5.00pm (AEST-Australian Eastern Standard Time), with voicemail after hours.

## Types of support

There are a number of ways of accessing support services.

### Online support

*USQAssist* is a web self-serve facility for you to:

- find answers to common questions at any time,
- ask any questions,
- track the progress of your question,

- keep a record of questions and responses.

To access *USQAssist* go to <http://usqassist.usq.edu.au> or click on “*USQAssist*” in *USQConnect*.

### **Telephone support**

If you prefer to telephone, call Outreach Services on (07) 46312285 for assistance. Outreach Services is staffed weekdays between 8.30am and 5.30pm (AEST), with voicemail after hours. If you are located in Eastern Australia, contact your Regional Liaison Officer.

International students telephone the USQ International Office on +61 7 46312362, or your Agent. USQ International is staffed weekdays between 9.00am and 5.00pm (AEST), with voicemail after hours.

### **Fax**

International students fax the USQ International Office on +61 7 46362211. All other students fax the Distance and e-Learning Centre on (07) 46361049.

### **Postal address**

The Administrator  
Distance and e-Learning Centre  
University of Southern Queensland  
Toowoomba Qld 4350  
Australia

## ***USQConnect***

*USQConnect* provides you with online access to information, services and course resources relevant to your studies from a convenient, central point. To access *USQConnect*, from the USQ home page at <http://www.usq.edu.au> click on *USQConnect*, or go directly to the URL at <http://usqconnect.usq.edu.au>. You will require your *USQConnect* username and password to access the system. You will be notified of this username and password on your first Letter of Offer or Enrolment Notice.

### **Study Desk**

Your *StudyDesk* in *USQConnect* gives access to a home page for every course in which you are currently enrolled. Content available from the course home page will vary according to the teaching requirements of the course, but may include:

- course materials and resources
- electronic discussion facilities
- access to past examination papers.

As each course has specific learning requirements, availability of these features will vary between courses.

### **USQAdmin**

USQAdmin, also accessed through USQConnect, allows you to access a number of administrative functions such as changing your contact details, checking your enrolment details, accessing learning circles, checking final grades, viewing your exam timetable, changing your exam centre, and more.

### **Other links**

USQConnect also gives access to the Library and the Academic Learning Support site, as well as the Quick Links list of University sections and services.

## **Lecturer's contacts**

Students should contact the lecturer weekly for homework discussions. At the time of writing the lecturer's contact details were:

Dr Sergey Suslov, Department of Mathematics and Computing, University of Southern Queensland, Toowoomba, Queensland 4350, Australia.

Email: [ssuslov@usq.edu.au](mailto:ssuslov@usq.edu.au),

Phone: +61-7-4631-5542, Fax: +61-7-4631-5550.

The most recent information on the teaching staff involved with the course will be available from the course web page at <http://www.sci.usq.edu.au/courses/mat3106>.

All external students enrolled in the course are required to contact the lecturer during the first week of semester to discuss their preferred regular communication options.

## Course evaluation

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Feedback from students studying USQ courses is very valuable to the university. Each semester, a number of courses are selected for student evaluation. If a course that you are studying is selected for evaluation you will receive an email message requesting your feedback. The email will include the link to the electronic survey form and give directions for its completion.

## Study Schedule, Semester 2, 2008

Week Dates	Study	Homework (Activity) <sup>1</sup>	Assignments <sup>2</sup>
<b>1</b> 21–25 July	§§1.1–1.3		
<b>2</b> 28 July–1 Aug	§§1.3–1.5	<b>1.C–1.G</b>	Question 1, Assignment 1
<b>3</b> 4–8 Aug	§§1.5, 1.6		Question 2, Assignment 1
<b>4</b> 11–15 Aug	§§2.1, 2.2	<b>1.I–1.R</b>	Question 3, Assignment 1
<b>5</b> 18–22 Aug	§§2.3, 2.4		Question 3, Assignment 1
<b>6</b> 25–29 Aug	§§2.4, 2.5	<b>2.B–2.O</b>	Question 3, Assignment 1
<b>7</b> 1–5 Sep	§§3.1, 3.2		<b>Assignment 1 due Sep 1</b>
<b>8</b> 8–12 Sep	§§3.3, 3.4	<b>2.Q–3.D</b>	Question 1, Assignment 2
<b>9–10</b> 15–26 Sep	Mid-semester break		Question 2, Assignment 2
<b>11</b> 29 Sep–3 Oct	§3.5, 4.1		Question 3, Assignment 2
<b>12</b> 6–10 Oct	§§4.1, 4.2	<b>3.F–3.L</b>	Question 4, Assignment 2
<b>13</b> 13–17 Oct	§§4.3.1		<b>Assignment 2 due Oct 13</b>
<b>14</b> 20–24 Oct	§5.1–5.2	<b>4.B–4.F</b>	
<b>15</b> 27–31 Oct	<b>Revision</b>	<b>5.B–5.E</b>	
<b>16–17</b> 3–14 Nov	Examination period		

<sup>1</sup> You must send the homework in on Mondays of the indicated weeks.

<sup>2</sup> Work on these Assignment questions during or before the indicated weeks.

## **Residential School**

There is no compulsory residential school for this course. External students who expect to be at USQ during the period of 15–26 September, 2008 and who would like to get help with their study of MAT3106 must contact the lecturer beforehand so that a mutually convenient consultation time can be arranged.

## Homework and Assignments

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In this course, regular homework plays a crucial role in both ensuring your firm understanding of the material and passing the subject. It contains exercises listed in the Activity sections in the Study Book which you have to attempt and submit to the lecturer for feedback on a regular basis (see the Study Schedule above). These exercises are not designed to check what you have learnt on your own (this will be the purpose of Assignments), but rather to help you in identifying the difficulties you have with the course material and in resolving them via immediate communication with the lecturer. In addition, some of the homework exercises contain important concepts which complement the lecture material and are used in the subsequent lectures/chapters. For this reason it is most important that you **follow the homework submission schedule consistently throughout the whole semester and start working on the homework problems from the very first day of studies**. Failure to do so will inevitably lead to continuously growing study difficulties and might have a disastrous effect on your overall result (remember that this course is offered every second year and failing it might have severe consequences in terms of your graduation date). The importance of homework is emphasised by its heavy contribution to your final grade, see Course Specifications. At the same time every student is guaranteed to get full marks for their homework provided all submission deadlines are met and any difficulties encountered are reported and discussed with the lecturer before the submission deadline. Upon contacting the lecturer with a homework question you will get a hint sufficient enough to complete the problem with no error. Every correctly solved homework problem including those you requested help with will add 3 points towards your homework score. This is a guaranteed opportunity for every student in the class to earn 15% towards their final grade with the only condition that any difficulties must be identified and resolved with the lecturer **before the specified deadlines**. Do not neglect this chance! In order to accelerate receiving the lecturer's feedback, external students who choose to mail their homework solutions rather than email or fax them should post their submissions directly to the lecturer (see "Lecturer's contact" section) and not to DeC.

In contrast to the homework, you are expected to work on the assignment problems on your own. They are supposed to check how successful you are in using the information obtained from the homework discussions. External

students should submit their Assignment solutions through DeC. In order to help you to manage your time the Study Schedule suggests the time frames over which you should be working on the specific Assignment questions.

## Requirements for presentation of your assignments and homework

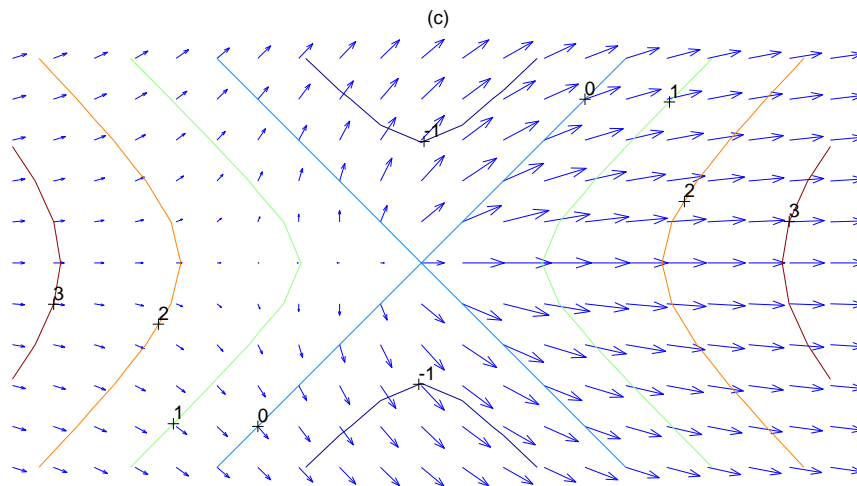
- Submit the solutions in the order corresponding to the questions in the assignment/homework. Number the question and all its items in your solutions. If you are not able to provide solutions for some questions or items indicate this clearly by, leaving a blank space after the question/item number.
- Write (or type) your solution results neatly. Give clear references to any plots used.
- Plots must include titles and appropriate labels (if you use MATLAB type `help title`, `help xlabel` or `help gtext`, respectively). All attached plots *must* be referenced and discussed in your solutions. Plots which are not labelled, not referenced and/or not discussed might be discarded by the lecturer as irrelevant.
- While we encourage group work in the resolution of difficulties in the course material, *in the assignments the solutions must be results of your work alone*. In the case of a group of students being identified as submitting identical or very similar solutions, the examiner may assign a single mark to the whole group so that the individual mark will be the group mark divided by the number of group members.

# MAT3106 Assignment 1

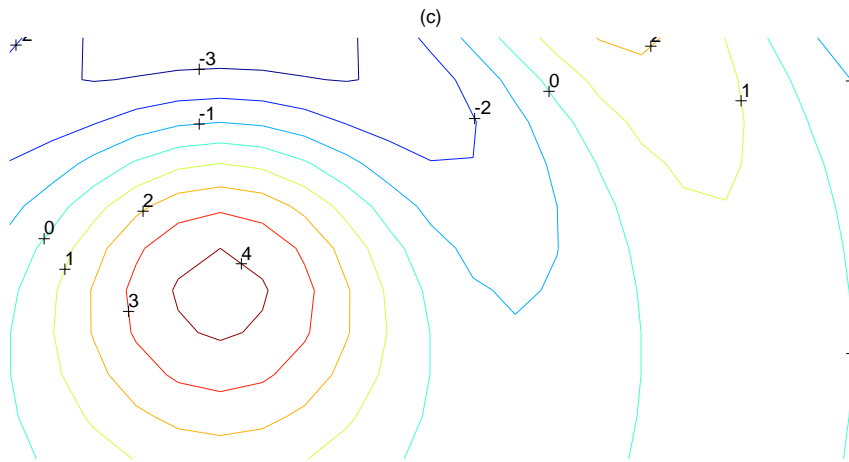
Due Date September 1, 2008, Weight 15%

## Question 1.

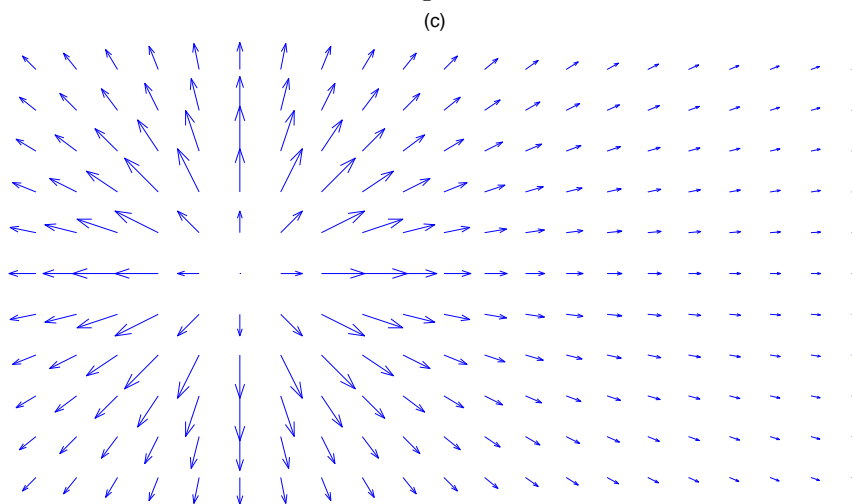
- (a) Given the plotted level curves of a scalar steady function  $f$  and the velocity field  $\vec{v}$ , sketch the regions where the material derivative  $Df/Dt$  is positive, negative and approximately zero respectively. How would your answer change if  $f$  was time dependent?



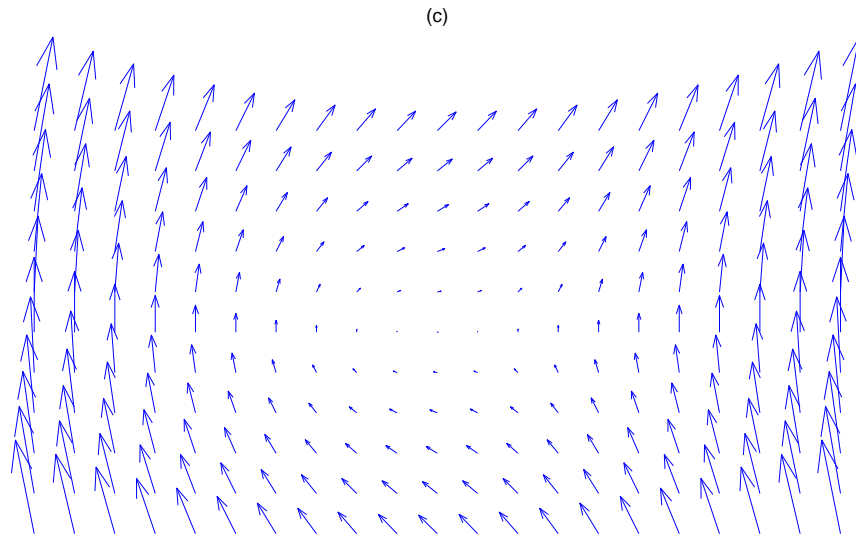
- (b) For the scalar function whose level curves are plotted below, sketch in the vector field of its gradient—endeavour to get the directions and relative magnitudes correct.



- (c) The vector field shown below represents the velocity field of a compressible fluid. By roughly estimating the sign of the divergence, mark regions in which you expect the density to be increasing in time and in which it would be decreasing.



- (d) For the following velocity field, indicate regions where you expect the vorticity to be positive, negative, and approximately zero.



**Question 2.**

The velocity potential for a two-dimensional fluid flow is  $\phi = (-3x + 4y) \cos t$ .

- (a) Is the flow irrotational?
- (b) Is the flow divergence free?
- (c) If so, determine the pressure field.
- (d) Derive an algebraic expression for a streamfunction.
- (e) Derive an algebraic expression for a streamline through point (1,1).
- (f) Find the flow rate through the line connecting points (1,1) and (1,2). Show details.
- (g) Does the fluid particle moving with the flow away from point (1,1) at time  $t = 0$  experience increasing or decreasing pressure? Justify your answer in detail mathematically.
- (h) Does the fluid particle in the previous item move in the direction of increasing or decreasing velocity potential? Explain in detail.

**Question 3.**

Do questions 8, 14, 24 and 34 from Kreyszig's 9th edition CHAPTER 10 REVIEW, PP. 473–474.

# MAT3106 Assignment 2

Due Date October 13, 2008, Weight 15%

## Question 1.

Solve item (i) in Problem 10 on page 18 in Ockendon & Ockendon. Note that symbol  $\wedge$  denotes vector product and is equivalent to symbol  $\times$ . You may wish to use the vector identities given in the Appendix in the Study Book to solve this problem. However you are required to reference every identity you use by its number in the Appendix.

**Bonus:** solve item (ii) to get extra points towards your assignment score.

## Question 2.

The differential equation of salt conservation for sea water flowing with velocity  $\vec{v} = (u, v, w)$  is

$$\frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} + w \frac{\partial S}{\partial z} = \kappa \left( \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2} \right),$$

where  $\kappa$  is a (constant) coefficient of diffusion and  $S$  is the salinity in parts per thousand. What are the dimensions for  $\kappa$ ? Nondimensionalise the above equation and discuss any nondimensional parameters which appear. Consider  $\kappa = 0$  and discuss how the above equation is related to Exercise 1.19 in your study book.

## Question 3.

Solve Problem 13 on page 19 in Ockendon (only the first paragraph in the problem definition). Note that the term “cylindrical pipe” does not necessarily mean that a pipe is of circular cross-section and  $c = \frac{\partial p}{\partial x}$ .

## Question 4.

Solve Problem 9 on page 46 in Ockendon.