

MAT3101

Mathematical Modelling

Faculty of Sciences

Introductory Book

Semester 1, 2003

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Course Specifications

Resource Materials

Study Books

The study guide is very essential for this course as it contains a large part of the study material (especially for Strand II). There are also text books to be purchased for this course which are to be used in conjunction with the study book. Please see the course specification for details of the textbooks.

Software

Access to MATLAB software is desirable in order to successfully complete this course, but other similar packages with built in graphics can be used as an alternative. Students not having access to any of such packages should contact the examiner to discuss their options.

Calculator

Any scientific calculator should suffice.

Student Enquiries

You should carefully read the information provided in your *USQ Student Guide* concerning contact details and support services.

Enquiries via the Internet

If you have Internet access, **USQ Assist** is the most efficient method for requesting support assistance. This is a web self-service facility for **all** students. You can:

- find answers to common questions;
- ask a question; and/or
- track the progress of a question.

By typing a keyword in the search field, you can find answers to many of the questions frequently asked by students, including course troubleshooters. To access USQAssist, go to <http://usqassist.usq.edu.au>.

or click on *Assist* in *USQConnect*.

Enquiries via Telephone or Facsimile

Alternatively, you can ask for support via telephone or fax.

| | International Students | All Other Students |
|-------------------|---|---|
| | Contact your Local Support Office for further assistance. If there is no Local Support Office in your country you should contact the International Office at USQ. | All administrative queries should be directed to the Outreach Services in the Distance Education Centre (DEC) or your Regional Liaison Officer. |
| Telephone: | 61 7 46312362 (International Office) | 07 46312285 (Outreach Services) |
| Fax: | 61 7 46362211 (International Office) 61 7 46359225 (Office) | 07 46361049 (Outreach Services) |
| Web Form: | http://usqconnect.usq.edu.au/usqassist | |
| Email: | iosupport@usq.edu.au | outreach@usq.edu.au |

USQConnect

USQConnect provides you with online access to information, services and course resources relevant to your studies from a convenient, central point. This integrated approach means you are assured of wide-ranging, up-to-date information via a consistent and easy-to-use interface. To access USQConnect, go to the USQ Homepage at <http://www.usq.edu.au> and click on USQConnect, or go directly to the USQConnect URL at <http://usqconnect.usq.edu.au>. You will require a USQConnect username and password to access the system. You will be notified of this username and password by Student Administration on your first Enrolment Notice and Statement of Account.

In USQConnect, your **USQ Study Desk** gives access to:

- electronic course materials and resources (where appropriate); and
- electronic course discussion facilities.

As each course has specific learning requirements, availability of these features will vary between courses.

Note: Up-to-date information about using the Study Desk is available on the USQConnect homepage beside the Study Desk link.

USQAdmin allows you to access a number of student administration functions such as changing your contact details, checking your enrolment details, accessing Learning Circles, checking assignment results and final grades, changing your exam centre, and more.

USQConnect also gives access to the Library and the Academic Support site, as well as the Quick Links list of University sections and services.

Lecturer contact

The lecturer may be contacted directly by emailing to Dr S.A. Suslov at ssuslov@usq.edu.au

Course evaluation (External Students Only)

The University of Southern Queensland is committed to continuous improvement, and **seeks your input** to that process through your participation in our course evaluation process. Please complete and return the questionnaire Student Feedback on External Courses included later in this introductory book.

Your response will be processed so that, unless you wish otherwise, the course Examiner will not be aware of your identity. Please help us to help our students by providing feedback on your experiences in this course.

When to Return the Questionnaire

Please return the questionnaire before the end of this semesters examination period.

Where to Send the Questionnaire

- (a) Insert the completed questionnaire in an envelope, seal and address envelope as follows:

*The Course Evaluation Co-ordinator
Information Technology Services
University of Southern Queensland
Toowoomba 4350*

- (b) The envelope may be posted directly to the above address

OR

attached to the outside of your last assignment for this course and then posted to DEC.

Study Schedule, Semester 1, 2003

| Week/Dates | Strand I | Strand II | Homework ¹ | Assessment |
|----------------------------------|------------------------|------------------------|---|--|
| 1 3–7 March | S, Ch. 1, Sec. 1–7 | Sec. II.1.1, II.1.2 | | |
| 2 10–14 March | S, Ch. 2, Sec. 1, 2 | Sec. II.1.3, II.1.4 | Str. I, Mod. 1 | |
| 3 17–21 March | S, Ch. 2, Sec. 2, 3 | Sec. II.1.5 | Str. II, Mod. 1, Q. 1.1–1.16 | |
| 4 24–28 March | S, Ch. 3, Sec. 1, 3 | Sec. II.2.1, II.2.2 | Str. I, Mod. 2 | |
| 5 31 March– 4 April | S, Ch. 3, Sec. 3, 4 | Sec. II.2.3, II.2.4 | Str. I, Mod. 3; Str. II, Mod. 1, 2, Q. 1.17–2.4 | Assnt 1, Str. I & II due 4 April |
| 6 7–11 April | S, Ch. 4, Sec. 1 | Sec. II.2.5 | | |
| 7 14–18 April | S, Ch. 4, Sec. 2 | Sec. II.3.1 | Str. I, Mod. 4 | |
| 8–9 21 April–2 May | Mid-semester break | | | |
| 10 5–9 May | S, Ch. 5, Sec. 1 | Sec. II.3.2 | Str. II, Mod. 2, 3, Q. 2.5–3.3 | Assnt 2, Str. I due 9 May |
| 11 12–16 May | S, Ch. 5, Sec. 1, 2 | Sec. II.3.3 | Str. I, Mod. 5 | |
| 12 19–23 May | S, Ch. 5, Sec. 2 | Sec. II.4.1, II.4.2 | Str. II, Mod. 3, Q. 3.4–3.12 | |
| 13 26–30 May | S, Ch. 8, Sec. 1 | Sec. II.4.3 | Str. I, Mod. 6, 7 | Assnt 3, Str. I due 30 May |
| 14 2–6 June | S, Ch. 9, Sec. 1 | Sec. II.4.4.1 | Str. II, Mod. 4, Q. 4.1–4.14 | |
| 15 9–13 June | S, Ch. 9, Sec. 1 | Sec. II.4.4.2 | Str. I, Mod. 8 Str. II, Mod. 4 Q. 4.15–4.19 | |
| 16–18 16 June–4 July | Examination period | | | |

¹ External students must send the homework in on Mondays of the indicated weeks. Daytime students must submit homework weekly.

Residential School

There is no compulsory residential school for this course. External students who expect to be at USQ during the period of 21 April–2 May and who would like to get help with their study of MAT3101 must contact the examiner beforehand so that a mutually convenient consultation time can be arranged. External students visiting the USQ campus may wish to watch the videos recommended for Strand II at the USQ Library (without borrowing the films) as they are very nice illustrations for the course material.

Assignments and Homework

In this course, weekly homework plays a crucial role in both ensuring your firm understanding of the material and passing of the subject. You *must* solve *all* homework problems and submit them weekly. The homework for Strand I consists of two parts:

- (a) Your notes taken while reading the set text. It should show working of the mathematics between the displayed equations to demonstrate that you have followed the reasoning. The notes need only be brief. Aim to write your reports in such a way that in case you need to find a reference to a specific concept or part of the material sometime after you finished the study your report should be a straightforward guide with major concepts, methods, definitions and conclusions listed. Answer all control questions listed in the Study Book for each topic.
- (b) The exercises embedded in the text. You will receive full marks for a reasonable attempt, not necessarily for a complete answer—some can be quite challenging. The lecturer will be happy to discuss these with you before you submit them.

Strand II homework consists of questions found at the end of each section in the study book. It is very similar to assignments (and is heavily weighted) with the only difference that you are encouraged to contact the lecturer for detailed discussions on any difficulties you encounter when solving the homework problems. Homework problems are chosen in such a way that they *complement and extend* the lecture material, not simply repeat it, and, most importantly, their solutions are frequently used as starting points for the next lectures. Thus Strand II homework is an integral part of the lecture material and is absolutely compulsory. Remember that its main purpose is not the assessment mark but ensuring your good understanding of the material and you should contact the lecturers to resolve any difficulties promptly.

In contrast to the above, it is expected that you work on assignments independently, contacting the lecturer only in exceptional cases. Also keep in mind that unlike in many other courses you have taken so far the weight of all assignments, homework and final examination/project here are approximately equal. This means that you must work hard on studying the course material throughout the whole semester: you will not have a chance of catching up in the end of the semester if you did poorly in its beginning.

Requirements for presentation of your assignments and homework

- Submit the solutions for the problems in the order corresponding to the questions in the assignment. Start each new question on a separate sheet. Number the question and all its items in your solutions. If you are not able to provide solutions for some questions or items indicate this clearly by, for example, leaving a blank space after the question/item number.
- Write (or type) your description of the method used and the results obtained (and any conclusions reached if appropriate). In your description, give clear references to any programs or plots used.
- Include a printout of all programs (with proper comments) and relevant program output if applicable.
- Include plots of any relevant graphs. Plots must include titles and appropriate labels (if you use MATLAB type `help title`, `help xlabel` or `help gtext`, respectively). All attached plots *must* be referenced and discussed in your solutions. Plots which are not labelled, not referenced and/or not discussed will be discarded by the assignment marker as irrelevant.
- While we encourage group work in the resolution of difficulties in the course material, *in the assignments the solutions must be results of your work alone*. In case a group of students is identified submitting identical or very similar solutions, the marker may assign a single mark to the whole group so that the individual mark will be the group mark divided by the number of group members.

MAT3101 Assignment 1, Strands I and II

Due Date April 4, 2003, Weight 30%

Strand I:

Question 1.

Svobodny, Chapter 1, Problem 3, page 29

Question 2.

Svobodny, Chapter 2, Exercises 3, page 38 and Problem 6, pages 56–57

Question 3.

Svobodny, Chapter 2, Exercise 13, page 53

Strand II:

Question 4.

Review the appropriate material you studied in MAT2102 Vector Calculus and Partial Differential Equations. Then

- Write the Laplace equation $\nabla^2\phi = 0$ for the flow potential ϕ in polar coordinates (r, θ) .
- Use the method of separation of variables, i.e. write

$$\phi(r, \theta) = R(r)F(\theta), \quad 0 \leq \theta \leq 2\pi$$

to solve the Laplace equation and show that its general solution is

$$\begin{aligned} \phi = & c_0 + (c_1 + c_2\theta) \ln r \\ & + \sum_{n=1}^{\infty} \left[\left(A_n r^n + \frac{B_n}{r^n} \right) \cos(n\theta) + \left(C_n r^n + \frac{D_n}{r^n} \right) \sin(n\theta) \right]. \end{aligned} \quad (1)$$

- (c) Write general polar coordinate expressions for the flow velocity components u_r and u_θ in terms of ϕ , then substitute (1) into them.
- (d) Consider the flow of inviscid incompressible fluid with density ρ around a circular solid cylinder of radius a . Specify the velocity boundary condition at the surface of the cylinder. Assume that the flow far away from the cylinder is uniform with a horizontal velocity u_∞ . Then the boundary conditions at $r \rightarrow \infty$ are

$$u_r = u_\infty \cos \theta, \quad u_\theta = -u_\infty \sin \theta$$

(you will get bonus points if you show this). Use these boundary conditions and results of item c to determine the coefficients in (1). Write the final expressions for the velocity components u_r and u_θ .

- (e) Finally use the Bernoulli equation to derive the expression for the pressure distribution along the surface of the cylinder.

Question 5.

Solve Problem 18, page 21 in Ockendon & Ockendon. Derive the expressions for streamlines for this flow and explain why this flow is called the stagnation point flow. What is the flow rate through the surface containing points $(x, y) = (-2, 1)$ and $(x, y) = (-1, 2)$?

Question 6.

The three-dimensional irrotational incompressible flow is given by the following velocity field

$$\vec{V} = [x^2 - y^2, v(x, y, z), 2yz] .$$

Find the most general expression for the velocity component $v(x, y, z)$.

MAT3101 Assignment 2, Strand I

Due Date May 9, 2003, Weight 15%

Question 1.

Svobodny, Chapter 3, Exercise 2, page 67 and Exercise 5(i–iii), page 70

Question 2.

The equation describing evolution of a fluid flow vorticity ω in the polar coordinates (r, θ) is

$$\frac{\partial \omega}{\partial t} = \nu \left[\frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} \right],$$

see Section 2.3 in Strand II of the MAT3101 Study Book. Use the Pi-procedure and the Pi-theorem to suggest a possible functional form of the solution for ω . You do not need to solve the above equation. Compare your derived solution form with that of equation (2.8) in the Study Book, discuss and justify any differences.

Question 3.

Svobodny, Chapter 4, Exercise 8, page 108

Question 4.

Svobodny, Chapter 4, Problem 4, page 130

MAT3101 Assignment 3, Strand I

Due Date May 30, 2003, Weight 15%

Question 1.

Svobodny, Chapter 5, Exercise 3, page 148 and Exercise 6 (i, ii), page 152

Question 2.

Svobodny, Chapter 5, Exercise 17, page 180 and Exercise 18, page 182

Question 3.

Svobodny, Chapter 8, Exercise 4, page 309