

Recursive Functions

- Describe what the following function does by completing the comment lines.

```
[0] p = Q(N)
[0.1]NB. Input N is a list of real numbers
[0.2]NB. Output p is ...
[0.3]NB. For example Q(16) = ...; Q(32, 5) = ... and Q(8, 27, 125, 3) = ...
[1] If length(N) = 1
    [1.1] p ← Q(N, 2)
    else
    [1.2] p ← curtail(N) ∧ (1/last(N))
```

Here the *power* function, \wedge , is assumed to work term by term on every element of a list so that, for example, $(3, 4, 5) \wedge 3$ returns 27, 64, 125.

What does $Q(23.56, 38.37, 126.193, 2.25) \wedge 2.25$ return?

- Write pseudo-code for a recursive function that, given a list, returns the list in reverse order.
- The following is an algorithm for McCarthy's function.

```
[0] p = M(n)
[1] If n > 100
    [1.1] p ← n - 10
    else
    [1.2] p ← M(M(n + 11))
```

Working in groups of four or five collect data on the output of this function.

Clearly the function is well-defined for $n > 100$, but do you think it is well defined for positive integers less than 100? What is its range of values for integers n in $1 \leq n \leq 100$?

- Consider the following function for the n^{th} term of a sequence.

```
[0] an = T(n)
[1] If n < 2
    [1.1] an ← 1
    else
    [1.2] an ← T(n - 1) + T(n - 2)
```

Use this algorithm to compute the first 6 terms of the sequence.

Write two algorithms that return the first n terms of the sequence, one that is recursive and another that computes the values iteratively.

Karnaugh Maps

1. Write the symbolic logic expression

$$(\neg x \wedge y \wedge z) \vee (x \wedge y)$$

in Boolean notation and draw a digital circuit for it. Write the disjunctive normal form for the expression; find a suitable reduction using a Karnaugh map and draw the corresponding digital circuit for the result.

2. Find the minterms for the following function and draw a digital circuit with the least number of components for it

a	b	c	$F(a, b, c)$
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1
