Truth Tables

Construct the truth table for each of the following expressions.

(i) $(p \wedge \neg q) \wedge (\neg p \vee q)$

p	$\neg p$	q	$\neg q$	$p \wedge \neg q$	$\neg p \vee q$	$(p \wedge \neg q) \wedge (\neg p \vee q)$
0	1	0	1	0	1	0
0	1	1	0	0	1	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0

This proposition is a contradiction. It can easily be seen from a venn Diagram which shows that the proposition asks for the intersection of two disjoint sets.

(ii) $p \vee \neg(\neg p \rightarrow q)$

p	$\neg p$	q	$\neg p \rightarrow q$	$\neg(\neg p \rightarrow q)$	$p \vee \neg(\neg p \rightarrow q)$	$q \rightarrow p$
0	1	0	0	1	1	1
0	1	1	1	0	0	0
1	0	0	1	0	1	1
1	0	1	1	0	1	1

The table shows that the simplest form of the proposition is $q \rightarrow p$.

(iii) $(p \rightarrow q) \wedge \neg q) \rightarrow \neg p$

p	$\neg p$	q	$\neg q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \neg q$	$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$
0	1	0	1	1	1	1
0	1	1	0	1	0	1
1	0	0	1	0	0	1
1	0	1	0	1	0	1

This is a tautology. it is easily interpreted because it says "if p implies q and it is not q, then it cannot be p" – which is true no matter what p and q are. A Venn Diagram will show that the entire region is covered.

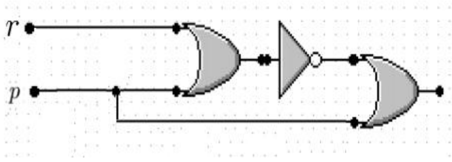
Logical expressions and Reductions

(a) $p \equiv$ *Peter enjoys the show*
$r \equiv$ *Rebecca enjoys the show*
$q \equiv$ *Peter enjoys the show or neither of them do.*

(i) write an algebraic expression, q.

$$q \;\equiv\; p \vee \neg(p \vee r)$$

(ii) Draw diagram with logic gates to describe the situation.



(iii) Make a table of all possible combinations of values for p and r and use your algebraic expression to calculate the corresponding values for q.

p	r	$p \vee r$	$\neg(p \vee r)$	q	$\neg p$	$\neg p \wedge r$	m
0	0	0	1	1	1	0	1
0	1	1	0	0	1	1	0
1	0	1	0	1	0	0	1
1	1	1	0	1	0	0	1

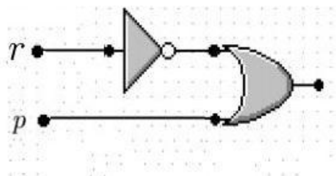
(iv) Express the proposition that it is not true that Peter will not enjoy a show when Rebecca does in terms of the propositional functions and check the values of this expression (m) for all possible values of (p, r).

$$m \equiv \neg(\neg p \wedge r)$$

(v) Compare the values of q and m.

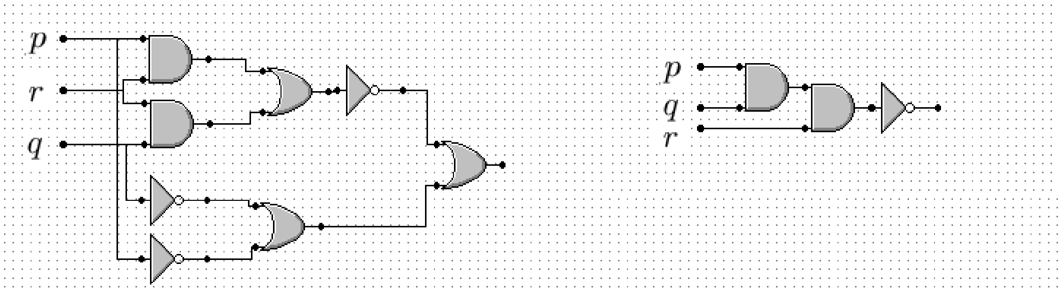
Truth values for q and m are shown in the table above and they are obviously equivalent propositions and are also equivalent to $r \to p$. This can be seen from an application of the de Morgan laws to the two expressions.

$$\begin{aligned} q &\equiv p \vee \neg(p \vee r) \\ &\equiv p \vee (\neg p \wedge \neg r) \\ &\equiv (p \vee \neg p) \wedge (p \vee \neg r) \\ &\equiv t \wedge (p \vee \neg r) \\ &\equiv p \vee \neg r \quad \equiv \quad r \to p \\ &\equiv \neg(\neg p \wedge r) \quad \equiv \quad m \end{aligned}$$



(b) Simplify the following expressions. Draw logic circuit for both the original and final expressions.

(i) $$\begin{aligned} ((p \wedge r) \vee (q \wedge r)) \rightarrow (p \rightarrow \neg q) &\equiv ((p \vee q) \wedge r) \rightarrow (\neg p \vee \neg q) \\ &\equiv \neg((p \vee q) \wedge r) \vee (\neg p \vee \neg q) \equiv \neg(p \vee q) \vee \neg r \vee \neg(p \wedge q) \\ &\equiv \neg((p \vee q) \wedge (p \wedge q)) \vee \neg r \equiv \neg((p \wedge q) \vee \neg r \\ &\equiv \neg(p \wedge q \wedge r) \end{aligned}$$

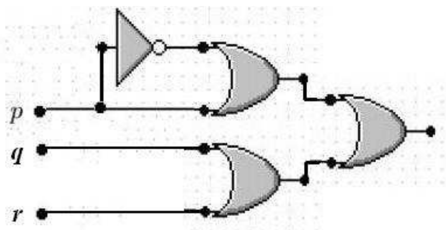


(ii) $p \rightarrow (q \rightarrow r) \leftrightarrow (p \wedge q) \rightarrow r$

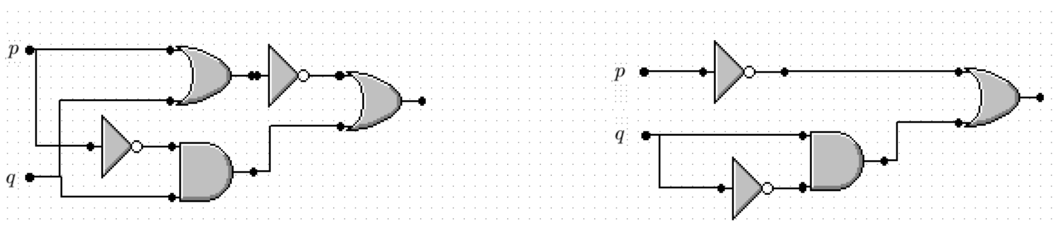
The result of an equivalence test is either true or false.

$p \rightarrow (q \rightarrow r) \equiv \neg p \vee (\neg q \vee r)$
$(p \wedge q) \rightarrow r \equiv \neg(p \wedge q) \vee r \equiv \neg p \vee \neg q \vee r$

Consequently $[p \rightarrow (q \rightarrow r) \leftrightarrow (p \wedge q) \rightarrow r] \equiv t$

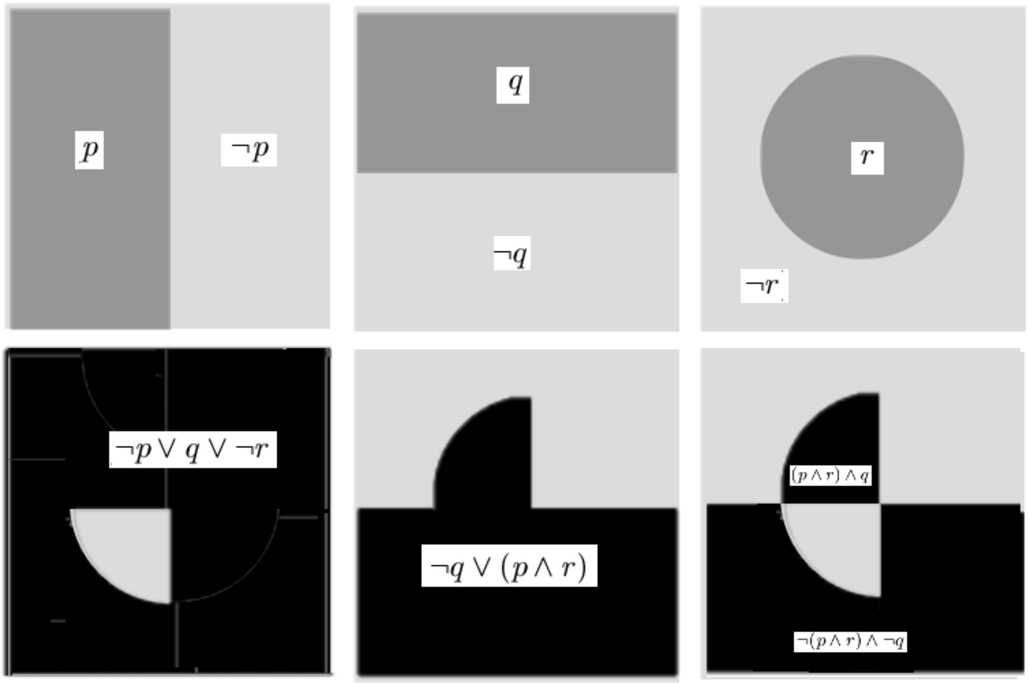


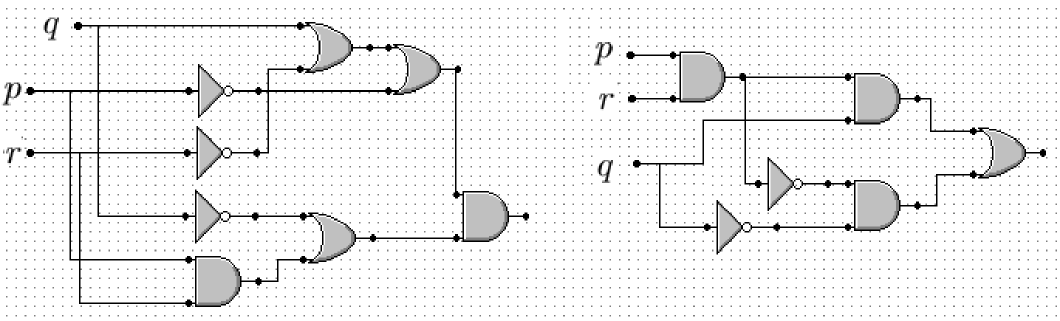
(iii) $\neg(p \vee q) \vee (\neg p \wedge q) \equiv (\neg p \wedge \neg q) \vee (\neg p \wedge q) \equiv \neg p \wedge \neg(\neg q \vee q) \equiv \neg p$



(iv) $$\begin{aligned} (p \rightarrow (q \vee \neg r)) \wedge (q \rightarrow (p \wedge r)) &\equiv (\neg p \vee q \vee \neg r) \wedge (\neg q \vee (p \wedge r)) \\ &\equiv (\neg(p \wedge r) \vee q) \wedge ((p \wedge r) \vee \neg q) \\ &\equiv [\ (p \wedge r) \wedge q\ ] \vee [\ \neg(p \wedge r) \wedge \neg q\ ] \end{aligned}$$

The final step may be obtained by applying the distributive law or can be seen from Venn Diagrams (several replicas are drawn for the sake of clarity:


p
$\neg p$
q
$\neg q$
r
$\neg r$
$\neg p \vee q \vee \neg r$
$\neg q \vee (p \wedge r)$
$(p \wedge r) \wedge q$
$\neg(p \wedge r) \wedge \neg q$


q
p
r
p
r
q