

Truth Tables

Construct the truth table for each of the following expressions.

(i) $(p \wedge \neg q) \wedge (\neg p \vee q)$

(ii) $p \vee \neg(\neg p \rightarrow q)$

(iii) $(p \rightarrow q) \wedge \neg q \rightarrow \neg p$

Find the simplest expression for propositions (i), (ii) and (iii) above.

Logical expressions and Reductions

- (a) Peter and Rebecca often go to the movies together and they either enjoy them or they don't but it has been observed that Peter is pretty easy to please and that either he enjoys the show or neither of them do.

There are two propositions here, one is that *Peter enjoys the show* and the other is that *Rebecca enjoys the show*. Writing p and r to represent the propositional functions for these two propositions,

- (i) write an algebraic expression, q , using the classical logical symbols \vee, \wedge and \neg to express the situation described.
 - (ii) draw diagram with logic gates to describe the situation.
 - (iii) Make a table of all possible combinations of values for p and r and use your algebraic expression to calculate the corresponding values for q .
 - (iv) It seems likely that if Rebecca enjoys a show then so will Peter. Express the proposition that it is not true that Peter will not enjoy a show when Rebecca does in terms of the propositional functions and check the values of this expression (m) for all possible values of (p, r) .
 - (v) Compare the values of q and m .
- (b) Simplify the following expressions, *i.e.* find equivalent propositions involving fewer operations. You may use the laws of logic, truth table or Venn diagrams or a combination of these approaches to find your final answer. Draw logic circuit for both the original and final expressions.

(i) $((p \wedge r) \vee (q \wedge r)) \rightarrow (p \rightarrow \neg q)$

(ii) $p \rightarrow (q \rightarrow r) \leftrightarrow (p \wedge q) \rightarrow r$

(iii) $\neg(p \vee q) \vee (\neg p \wedge q)$

(iv) $(p \rightarrow (q \vee \neg r)) \wedge (q \rightarrow (p \wedge r))$
