# Tutorial 2 Week of 24 November, 2008

## Algorithms for Decimal to Binary Conversion

To convert number like 379.84 to it's binary representation, start with the largest power of 2 contained in the number. In this case it is $256 = 2^8$.

Instead of adding powers of 2 to the 256, subtract the 256 from the original number and look for the largest power of 2 in what is left over. In this case, we have 123.84 left over and the highest power of 2 in this is $2^6 = 64$.

Repeat the process over and over until the integer part of the number is accounted for. In this case we have left over, after 64 is removed, 59.84 and $2^5 = 32$ is the highest power of 2 contained in it. Taking away the 32 leaves 27.84. The highest power of 2 in this is $2^4 = 16$, leaving 11.84. Taking out $8 = 2^3$, then 2 ($= 2^1$) and 1 ($= 2^0$), we are finally left only with the fractional part, 0.84. We have

$$379 = 2^8 + 2^6 + 2^5 + 2^4 + 2^3 + 2^1 + 2^0$$

so the binary representation for 379 is 101111011.

Instead of taking out halves (0.5), quarters (0.25), eighths (0.125), sixteenths (0.0625) etc. from the fractional number – most inconvenient for working by hand – we can take the following approach. If a number has a half in it, then doubling it will give a number bigger than 1. For example 0.84 doubled is 1.68. If we remove the 1 than the 0.68 is twice what would be left over with the half removed.

Now, if $0.84 - 0.5 = 0.34$ has a quarter (0.25) in it then multiplying it by 4 will give a number bigger than 1. However, we already have doubled the 0.34 to get 0.68, so we need only double this to get 1.36 to see that in fact there is a quarter on top of the half (i.e. 0.75) in the 0.84. What is left over is 0.09 and to see if there is an eighth in this we multiply if by 8 to get 0.72. But notice that we already have 4 times the difference in 0.36 and we need only double it to get the same answer. So we can repeat this process to get as many binary places as we want.

To this point we have the expansion $0.84 = 0.110\ldots$. Carrying on with the same process, we get $2 \times 0.72 = 1.44$, so $0.84 = 0.1101\ldots$; then $2 \times 0.44 = 0.88$, so $0.84 = 0.11010\ldots$; $2 \times 0.88 = 1.76$, so $0.84 = 0.110101\ldots$; and so on.

After the next step we will obviously have

$$379.84 = 101111011.1101011\ldots$$

the truncated expansion is the exact representation for the number 379.8359375. How many binary places are needed depends on the accuracy that we want to achieve.

It is convenient to separate the integer and fractional conversions for hand computation and it would probably be helpful to set up the computation sequences in the form of tables.

[ Exercises overleaf ]

1. Convert each of the following binary representations to decimals.

    (i) 1101.101

    (ii) 111010011

    (iii) 0.00001110101

    (iv) 1101.001101

    (v) 111010011.00011010111

2. Find the binary representation for the following numbers and set out your computations in convenient tables.

    (i) 4825

    (ii) 0.8217

    (iii) $\frac{13}{37}$

    (iv) 81.65

    (v) 2101.0226