

<i>Student name:</i>	Barcode
<i>Student number:</i>	
University of Southern Queensland	
Faculty of Sciences	
<b>MAT1101</b> Discrete Mathematics for Computing	
End Semester Exam	
Assessment No. 3: External	
This examination carries 60% of the total assessment for this course.	
Examiner: W. Spunde; Moderator: H. Butler	
Examination date: February — Semester 3, 200?	
Time allowed: Perusal: Ten (10) minutes Working: Two (2) hours	
<p><b>Special Instructions</b></p> <p>This is a <i>restricted</i> examination.</p> <p>Calculators may be used. Programmable calculators are allowed.</p> <p>Students may <b>not</b> write in the examination booklet during perusal time but may make notes on the examination paper.</p> <p><i>All examination question papers must be submitted to supervisors at the end of every examination and returned to USQ. Students are not permitted to retain them.</i></p> <p>Each student is permitted to bring <i>one single A4 sheet</i> of paper on both sides of which may be written any information the student wishes to have at hand. Students may also bring writing and drawing implements including coloured pencils, compasses, protractors, rulers, erasers, etc. into the examination room.</p> <p>Print your name and Student Number in all spaces provided on the Examination Paper, the Examination Booklet (if applicable) and the Computer Marked Answer Sheet (if applicable).</p>	

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**Question 1.****25 marks**

- (a) The table below shows *decimal* equivalents of the ASCII codes for the printable characters from the first 128 standard symbols. Add the row number to the column number of any particular symbol to get its decimal equivalent. The decimal equivalent for a space is 32. 127 is the decimal equivalent of the ASCII code for null.

$r + c$	0	1	2	3	4	5	6	7	8	9
30				!	"	#	\$	%	&	'
40	(	)	*	+	,	-	.	/	0	1
50	2	3	4	5	6	7	8	9	:	;
60	<	=	>	?	@	A	B	C	D	E
70	F	G	H	I	J	K	L	M	N	O
80	P	Q	R	S	T	U	V	W	X	Y
90	Z	[	\	]	^	_	'	a	b	c
100	d	e	f	g	h	i	j	k	l	m
110	n	o	p	q	r	s	t	u	v	w
120	x	y	z	{		}	~			

4 marks

- (i) Find the character string represented by the (8-bit) ASCII code

00101110011000110110111101101101

4 marks

- (ii) Write down the (8-bit) ASCII code for M01

- (b) If every integer must be stored in 16 bits in a computer with the same number of negative and non-negative integers, find:

3 marks

- (i) the range of integers that can be represented;

3 marks

- (ii) the integers represented by

1111111111111111, 1111111111111000 and 0001000000100010.

- (c) If floating point numbers are represented in 16 bits, of which 7 are assigned to the characteristic and that the bias for a
- $k$
- bit characteristic is
- $2^{k-1} - 1$
- , find:

4 marks

- (i) the computer representation of
- $-5.45$
- .

3 marks

- (ii) the smallest positive real number that can be represented in this system.

4 marks

- (iii) the range of numbers represented by 1100100111010110

**Question 2.****25 marks**

5 marks (a) Describe, with examples, what is meant by an *in-fix* syntax for a function. What other forms of function syntax can be found in the mathematical literature? Give at least one example of each form.

(b) In a particular spreadsheet the syntax for the `ceiling` function is

$$N = \text{CEILING}(n, m)$$

where  $n$  is any real number and  $N$  is the smallest multiple of  $m$  larger than  $n$ .

1 mark (i) If  $h \leftarrow 23.47$ ,  $k \leftarrow 7$  and  $z \leftarrow \text{CEILING}(h, k)$ , what number is stored in the variable  $z$ ?

2 marks (ii) What numbers are returned by

$$0.1 * \text{CEILING}(23.47 - 0.5, 1) \quad \text{and} \quad 0.01 * \text{CEILING}(234.7 - 0.5, 1)$$

5 marks (iii) Using function `CEILING` in the algorithm, write pseudo code for a function that, given inputs  $n$  and  $m$  and the same syntax as `CEILING`, returns the value of  $n$  rounded to  $m$  decimal places.

2 marks (iv) Using your function from part (iii), write the instruction that will round 2.765001 to 2 decimal places. What number will it return?

(c) Starting with any positive fraction,  $f$  less than 1, the next number in a sequence is obtained by multiplying the difference between the last number in the list and the square of that number by 1.5. For example the next number after 0.4 will be  $0.36 = 1.5 * (0.4 - 0.16)$ .

Function `GROWTH` takes inputs  $f$  and  $n$  and returns the  $n^{\text{th}}$  number in the sequence (counting from 0).

8 marks (i) Illustrate the difference between an iteratively defined function and a recursively defined function by writing pseudo code for function `GROWTH` with *two* different algorithms, one iterative and one recursive.

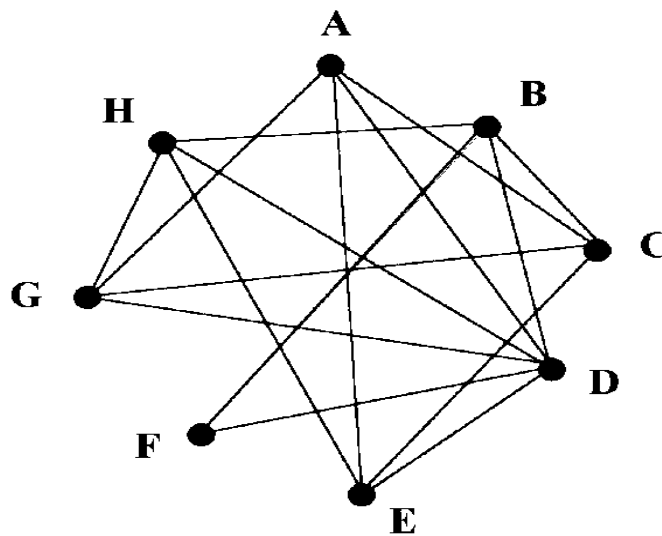
2 marks (ii) What are the essential features of any recursively defined function?

**Question 3.****25 marks**

- 3 marks (a) Determine which of the following statements is a proposition  
*“To be or not to be”*  
*“I think therefore I am”*  
*“I studied hard for this exam”*  
 and explain your reasons.
- (b) The statement,  $p$  :  
*“You cannot have satisfaction without commitment”*  
 is a compound proposition with component propositions  $q$  and  $r$  where  
 $q \equiv$  *“You have satisfaction”*  
 $r \equiv$  *“You have commitment”*
- 3 marks (i) Write  $p$  in terms of  $q, r$  and all or some of the operators  $\wedge, \vee$  and  $\neg$ .
- 3 marks (ii) Hence deduce that  $p$  can also be written as an implication.
- 2 marks (iii) Express the contrapositive of this implication verbally.
- 4 marks (iv) Write the converse and inverse implications in terms of the component propositions and express each in words.
- (c) Suppose that  $b$  represents any element of the set,  $\mathcal{B}$ , of all customers on a database and that  $g$  is any element of the set,  $\mathcal{G}$ , of all goods stocked by a supplier.  $P$  is a relationship between sets  $\mathcal{B}$  and  $\mathcal{G}$  defined by
- $b P g$  if, and only if, customer  $b$  has purchased item  $g$ .
- Interpret the symbolic statements below at left and, then complete the sentences to the right so that they have the same meaning. (Write your completed statements into your examination booklet.)
- 2 marks (i)  $\exists b \exists g : b P g$       *At least some of the goods ...*
- 2 marks (ii)  $\forall b \exists g : \neg(b P g)$       *Some of the goods ...*
- 2 marks (iii)  $\forall g \exists b : \neg(b P g)$       *There are customers ...*
- 4 marks (d) Prove that  $\forall n \geq 0 : 14^n - 27$  is divisible by 13.

**Question 4.****25 marks**

- 1 mark (a) What can be said about a connected graph with  $n$  vertices and  $n - 1$  edges, where  $n$  is a positive integer?
- 2 marks (b) What happens to the spanning tree of a graph if an edge (belonging to the graph) is removed from the tree? Explain your answer.
- (c) With reference to the graph below:

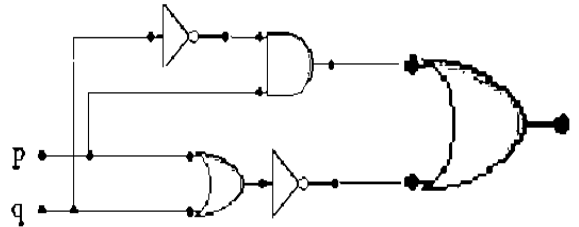


- 3 marks (i) Write down the adjacency matrix for this graph. Based on the analysis of your adjacency matrix, is this graph Eulerian? Explain your answer using the appropriate theorem.
- 2 marks (ii) If the graph is Eulerian, find an Eulerian circuit starting at vertex **B**. If not, adjust the graph by adding a suitable number of edges to make it Eulerian.
- 4 marks (d) Draw a weighted graph represented by the weight matrix

$$W = \begin{bmatrix} 0 & 11 & \infty & 5 & \infty \\ 11 & 0 & 7 & \infty & 10 \\ \infty & 7 & 0 & 9 & 13 \\ 5 & \infty & 9 & 0 & 8 \\ \infty & 10 & 13 & 8 & 0 \end{bmatrix}$$

and compute and sketch the minimal spanning tree.

(e) A diagram of a logic circuit is shown below.



Wires that cross without a dot do not touch.

2 marks

(i) Write a symbolic expression for the output of the circuit.

3 marks

(ii) Simplify the expression for the output and draw an equivalent circuit that produces the same output for inputs  $p$  and  $q$ .

(f) The truth table for a compound proposition with three component propositions  $a, b, c$  is shown below.

$a$	0	0	0	0	1	1	1	1
$b$	0	0	1	1	0	0	1	1
$c$	0	1	0	1	0	1	0	1
$g(a, b, c)$	0	0	0	1	0	1	0	1

3 marks

(i) Write down the disjunctive normal form for the proposition with truth values given by  $g$ .

5 marks

(ii) Draw the *simplest* logic circuit that produces output for inputs  $(a, b, c)$  identical to that of the propositional function  $g$ .

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End of the examination questions.

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